**FATA: An efficient optimization method based on geophysics**

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# Abstract

An efficient swarm intelligence algorithm is proposed to solve continuous multi-type optimization problems, named the fata morgana algorithm (FATA). By mimicking the process of mirage formation, FATA designs the mirage light filtering principle (MLF) and the light propagation strategy (LPS), respectively. The MLF strategy, combined with the definite integration principle, drives the algorithmic population to enhance FATA’s exploration capability. The LPS strategy, combined with the trigonometric principle, drives the algorithmic individual to improve the algorithm's convergence speed and exploitation capability. These two search strategies can better use FATA’s population and individual search capabilities. The FATA is compared with a broad array of competitive optimizers on 23 benchmark functions and IEEE CEC 2014 to verify the optimization capability. This work is designed separately for qualitative analysis, exploration and exploitation competence analysis, the analysis of avoiding locally optimal solutions, and comprehensive comparison experiments. The experimental results demonstrate the comprehensiveness and competitiveness of FATA for solving multi-type functions. Meanwhile, FATA is applied to three practical engineering optimization problems to evaluate its performance. Then, the algorithm obtains better results than its counterparts in engineering problems. According to the results, FATA has excellent potential to be used as an efficient computer-aided tool for dealing with practical optimization tasks. Source codes and related files are available at <https://aliasgharheidari.com/FATA.html> and other websites.

**Keywords:** Fata morgana algorithm; The mirage light filtering principle; Light propagation strategy; Swarm intelligence algorithm; Engineering optimization

# 2 Inspiration for the fata morgana phenomenon

The fata morgana (mirage) is a common physical phenomenon in nature. The mirage phenomenon formed by light propagation is the reflection of light from an object into an atmosphere of uneven density (from an optically denser medium to an optical thinning medium). By analyzing the phenomenon of mirages formed by the propagation of light rays emitted from underwater hills, this paper proposes the design of Figure 3, illustrating the optical propagation process of light rays emitted from a ship in the ocean that forms a mirage. Forming a mirage requires both an inhomogeneous density medium and light propagation in this medium. First, the atmospheric temperature will change because of the sunlight to form the inhomogeneous density medium. At this moment, the light reflected by the boat into this medium, the light in the propagation process, constantly changes the refraction angle, and eventually, the phenomenon of total internal reflection, the formation of the mirage. The mirage can be seen when the observer (Eye in Figure 3) looks at the sky in a red direction.



Figure 3. The formation of the mirage

According to the entire process of mirage phenomenon formation depicted in Figure 3, the light emitted from the ship's body can only form a mirage if it balances filtering the mirage light and light refraction/reflection operations during the propagation process. As mentioned earlier, there is currently an imbalance in the execution of population global search and individual local search strategies in swarm intelligence algorithms while searching for optimal values. Inspired by the balanced execution of filtering mirage light and light refraction/reflection operations in forming the mirage phenomenon, algorithms designed based on the mirage principle aim to balance global search and local search strategies during optimization, thereby demonstrating the best optimization capability. Instead of algorithms like the HHO, which simulate the global search and local search strategies of swarm intelligence algorithms using the soft and hard besiege strategies of hawk hunting, execute these strategies sequentially without maintaining a good balance.

Furthermore, based on the mirage principle, Figure 3 provides a detailed analysis of the ability of the algorithm to balance global search and local search strategies through the analysis of mirage light propagation. In the figure, when the object shoots light at the ship, some light will enter the atmosphere of inhomogeneous density. As the light propagates from an optically denser medium to an optically thinning medium in the atmosphere, the refractive index changes continuously to refract the light at an increasing angle. In the optical thinning medium, the light reaches the critical angle and undergoes the phenomenon of total internal reflection. Last, the mirage phenomenon is formed.

Therefore, incorporating the mirage principle into the design of swarm intelligence algorithms, this paper introduces the mirage light filtering principle and the light propagation principle based on the filtering of mirage light formation and the refraction and reflection operations of light, respectively. In the mirage phenomenon, the mirage light filtering principle can select light to form the mirage and filter out other light. The light propagation principle in a medium of inhomogeneous density can constantly change the direction and size of light.

Among them, the population search strategy of FATA (named the mirage light filtering principle) is inspired by the light reflected by the boat into the medium. The principle of light propagation inspires the individual search strategy of FATA (named the light propagation strategy) in the medium with inhomogeneous density. These two strategies are at the core of the FATA (fata morgana algorithm). FATA balances the mirage light filtering principle and the light propagation strategy responsible for global exploration and local algorithm exploitation. Therefore, the mirage formation process is entirely consistent with it, which creates the conditions for the proposed fata morgana algorithm.

# 3 Fata morgana algorithm

In Figure 4, multiple light that forms a mirage in the fata morgana algorithm is used as the population, while light () is used as the individual. The mirage () is used as the optimization target.

In the first stage, the multiple light population is dynamically evaluated according to the mirage light filtering principle based on the definite integral principle. The multiple light emitted from the hull in the lower left corner of Figure 4, including the light rays that have undergone physical transformation and formed a mirage (), and the light rays that have undergone physical transformation and are directed elsewhere without forming a mirage ().

In the second stage, the mirage light population executes the light propagation strategy (including the first half, the second half refraction strategy, and the total internal reflection). The physical change of light propagation in a medium with inhomogeneous density is the process of exchanging individual information, and the algorithm searches for the target to produce the mirage (optimal solutions).

## 3.1 The mirage light filtering principle



Figure 4. FATA optimization process in 3-dimension

The section shows the Fata Morgana algorithm’s population search strategy based on the principle of definite integral. In Figure 4, during the physical process of mirage formation, the hull emits two types of light rays. The majority of the light rays belong to the first type (other light in Figure 4), which do not propagate and form a mirage. The other type of light rays undergoes physical transformations that result in the formation of a mirage and are referred to as the mirage light ().

In FATA, distinguishing between the two types of light populations is crucial for the algorithm to find . Therefore, FATA employs a light population quality evaluation strategy based on the definite integral principle to assess the different types of light populations. In swarm intelligence algorithms, population quality is evaluated by calculating individuals' fitness and then aggregating the fitness values for the entire population. As shown in Figure 5a, if the fitness of individuals in a light population is ranked, it forms a cumulative curve. To efficiently compute the fitness of different types of light populations (other light, the mirage light), FATA utilizes definite integration to evaluate the curve in Figure 5b, using the integral value as a measure of fitness. The mirage light () that is selected based on the definite integral principle is also referred to as the filtered mirage light population.

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| --- | --- |
|  |  |
| (a) | (b) |

Figure 5. The population fitness curve of FATA

First, the strategy determines the population as other light, or the mirage light based on the population quality to perform different search methods (Eq. (1)). Population quality means the overall quality of the population. In the strategy, the integrated area () of the population fitness function () represents the population quality. Figure 5a shows the population fitness function curve. Figure 5b shows the integrated area () of the curve.

Fitness in the SIA represents individual quality. However, discrete and high-dimensional fitness values are difficult to use as an evaluation criterion for the population's overall quality. Therefore, all individual fitness in the population is fitted to a function (). Among them, the fata morgana algorithm is based on the principle of definite integration to calculate the integrated area () of the population fitness function curve.

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| --- | --- |
|  | (1) |
| (2) |
| (3) |
|  | (4) |
|  | (5) |

is the light individual. is the new individual. Algorithm 1 demonstrates the mirage light filtering principle of the fata morgana algorithm. Among them, Eqs. (2-3) are the first-half refraction strategy, the second-half refraction strategy, and the total internal reflection strategy, respectively (Section 3.2 will introduce them in that order). In Eq. (4), is the quality factor of the light population. The smaller the value of , the better the population's quality. represents the quality of the worst population. represents the quality of the best population. The mirage light populations have excellent population quality. In Eq. (5), is the individual quality factor. represents the fitness of the current individual (). represents the fitness of the worst individual. represents the fitness of the best individual.

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| **Algorithm 1** The mirage light filtering strategy |
| **Input:** light individual ;  Fit the population quality function according to the fitness of the individuals;  Calculate the integrated area of the based on the principle of definite integration;  Update the optimal area and the worst area ;  Calculate the population quality factor by Eq. (4);  **If**  The population is the light rays directed towards a medium with inhomogeneous density populations;  The population performs Eq. (1) to initialize the population randomly;  **Else**  The population is the light rays not directed towards a medium with inhomogeneous density populations;  The population executes the search strategy (Eqs. (2-3));  **End If**  **Return** new individual ; |

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| --- | --- |
|  | (6) |
|  | (7) |

Eqs. (6-7) show the method of calculating the area of the population fitness curve () based on the principle of definite integration. The principle of definite integrals uses the idea of the limit to calculate the area () of integration of . Eq. (6) is the population quality fitting function with points on the curve as and . and are parameters.

## 3.2 The light propagation principle

The light propagation principle in FATA is executed after the mirage light filtering principle, and it serves as the individual search strategy of the algorithm responsible for local exploitation in the search space to find local minima. As shown in Figure 6, the light population of FATA, represented by the mirage light rays, starts from the small boat in the lower-left corner. First, it undergoes the mirage light filtering strategy, where the light population is evaluated and filtered based on the principles of calculus to select the individuals that form the mirage phenomenon. Furthermore, the filtered mirage light population undergoes refraction and reflection sequentially. The individual changes in the light population during refraction and reflection can be observed in Figure 6. The light rays change in direction and size during the processes of refraction and reflection shown in the figure. As an individual search strategy, performing local exploitation in the search space to find a local minimum.



Figure 6. FATA is based on the mirage principle

The fata morgana algorithm designs the individual search strategy based on the light propagation principle combined with trigonometric functions. The algorithm chooses to execute the reflection strategy (the first half phase), the reflection strategy (the second half phase), and the refraction strategy based on the individual quality factor (in Eq. (5)).

**Light refraction (the first half phase).** In Figure 7, the light enters the medium with inhomogeneous density in the first half refraction, from optically denser medium to optical thinning medium propagation, changing the direction and size of the light. The angle of incidence () is smaller than the angle of refraction ().

Figure 7 analyzes the refraction process of the light individual. The light individual is . is the surface the refractive surface. In Eq. (8), is a new individual after the first half reflection strategy. Assume where is a constant. Eqs. (8-10) are the formulas for the strategy.

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| --- | --- |
|  | (8) |
|  | (9) |
|  | (10) |

is the new individual. is the current best individual. represents the refraction step of the strategy. is the first-half refraction ratio. is changing in the process of light propagation. In Eq. (10), to simply measure the incident angle () and the reflection angle () during refraction, the parameter replaces the angle change in the fata morgana algorithm, .

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| Figure 7. First refraction process of light | Figure 8. Second refraction process of light |

**Light refraction (the second half phase).** After performing the first half refraction phase, the light performs the second half refraction phase at random points. Figure 8 analyzes the second half refraction process of light. The angle of incidence is less than the angle of refraction . The light propagates in a medium with inhomogeneous density, so the refractive index () changes continuously. In the second half refraction strategy, the light individual () will generate a new individual () based on random individuals () in the search space. Eqs. (11-13) are the formulas for the strategy of FATA.

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|  | (11) |
|  | (12) |
|  | (13) |

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| --- | --- |
|  |  |
| (a) | (b) |

Figure 9. (a)Trends of (b) Trends of

is the refraction step in the second half refraction strategy. is a random individual from the population.  is the second refraction ratio. In Figure 9a, the value of oscillates randomly between  and gradually approaches zero as the increment of iterations. In Figure 9b, the value of oscillates randomly between [-150,150] and gradually increases with the number of iterations. It is found in Figure 9 that the values of the two parameters are relatively large. To standardize and , they are standardized. The strategy scales the two parameters to the interval. The substantial oscillation of at the last phase of the fata morgana algorithm enhances the ability to avoid the local optimum.



Figure 10. Total reflection process of light

**Light total internal reflection.** The total internal reflection phase is the final stage of light propagation in the formation of the mirage phenomenon. This is because as the refraction angle increases, the light undergoes total internal reflection in the medium with inhomogeneous density. The total internal reflection strategy drives the FATA population to explore in the opposite direction. Figure 10 analyzes the reflection process of light. The angle of incidence is equal to the angle of reflection . In the figure, is the center point of the interval (). and are the distances of the incident and refracted light to the horizontal plane, respectively. In the strategy, the light individual () is transformed into the individual () to search for the target in the opposite direction. Eqs. (14-17) are the formulas for the strategy of the fata morgana algorithm.

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| --- | --- |
|  | (14) |
|  | (15) |
|  | (16) |
|  | (17) |

is the reflected individual by the total internal reflection strategy. is the reflectance of the reflection strategy. controls the pattern of change in the light individual. When is greater than 1, crosses the boundary, . Therefore, the value of α will be discussed in section 4.2. represents the upper limit of the individual position. represents the lower limit of the individual position.

To further observe and analyze the algorithmic structure of FATA, the pseudocode of the entire FATA can be presented in Algorithm 2. Additionally, Figure 11 depicts the flowchart of the FATA, illustrating the optimization process of the two main population updating strategies in the FATA. The algorithmic structure of FATA mainly consists of population initialization, parameter initialization, and an iterative loop structure for the evolution strategy. Within the loop structure, the time complexity of the mirage light filtering principle and the light refraction strategy is primarily dependent on the number of iterations and is .

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| **Algorithm 2** Pseudocode of fata morgana algorithm |
| **Input:** parameters *, ,* ;  **Output:** best Individual;  Initialization parameters , , ;  Initialize a population of size ;  Calculate the fitness of each individual;  **While (**  update;  Calculate weights by Eq. (4);  Calculate and by Eq. (10) and Eq. (17);  **For** :  Execute Algorithm 1 to realize **the mirage light filtering principle**;  **If**  the light population performs Eq. (1) to initialize the population randomly;  **Else**  **If**  Update the individual by Eq. (8) according to **the first half light refraction strategy**;  **Else**  Update the individual by Eq. (11) according to **the second half light refraction strategy**;  Update the individual by Eq. (14) according to **the light total internal reflection strategy**;  **End If**  **End If**  **End For**  ;  **End While**  **Return** the best individual ; |



Figure 11. The flowchart of the fata morgana algorithm